

2.1 Derivative Basics: Form H Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_FJ.py`). All problems are original; provenance is in `PROVENANCE_GJ.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

Provided AS IS, without warranty of any kind; use at your own risk. Not every item has been vetted; problems, answers, and keys may contain errors.

General grading rules

- **Answer-only items** (Q1, Q9): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse), or a number from the wrong column of a table, earns 0 for that part unless a row below says otherwise.
- **Show-work items** (Parts II–IV other than Q9, and Part VI): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form H

Item	Answer	Rubric / trap
1(a)	2	<i>Trap:</i> the tangent drawn there invites the slope, 4; earns 0.
1(b)	2	<i>Trap:</i> giving the height $f(2) = -1$; earns 0.
1(c)	0	<i>Trap:</i> giving the height -2 of the low point; earns 0.
1(d)	DNE	Corner at $(-1, 2)$: one-sided slopes -1 and -4 . <i>Trap:</i> a slope of -1 or -4 , or deciding the derivative exists because $(-1, 2)$ is not a high or low point.
1(e)	$(1, 4)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-1, 4)$, which includes the falling stretch $(-1, 1)$.
1(f)	$f'(-3) < 0 < f'(2)$	$f'(-3) = -1$, $f'(2) = 2$. 3 pts, all or nothing.
2	$y = -3$	1 pt $g(1) = -3$; 2 pts $g'(x) = 6x^2 - 6$; 1 pt $g'(1) = 0$; 1 pt equation. $y - -3 = 0(x - 1)$ is accepted. <i>Trap:</i> a slope of 0 makes students think there is no tangent line, or they write the vertical line $x = 1$ (0 for the equation).
3	$y + 3 = 3(x + 1)$, i.e. $y = 3x$	1 pt $f(-1) = -3$; 2 pts $f'(x)$; 1 pt $f'(-1) = 3$; 1 pt equation in any correct form. <i>Trap:</i> $f(-1) = 1 - 3 - 1 = -3$ and $f'(x) = 4x^3 - 6x + 1$, $f'(-1) = -4 + 6 + 1 = 3$: signs at a negative input. Writing $x - 1$ for $x - (-1)$ earns 3 of 5.
4(a)	2	3 pts. Simplifies to $y = 2x - 4$. <i>Trap:</i> 6 (the coefficient before dividing by 2) or 3; earns 0.
4(b)	0	2 pts. Substitute $x = 2$. <i>Trap:</i> 2, from $2y - 4 = 0$ read as $y = 2$, or 4; earns 0.
5	6	<i>Trap:</i> p' is printed and invites $p'(3) = 11$; earns 0.

Item	Answer	Rubric / trap
6	$f'(x) = \frac{10}{(1-2x)^2}$	2 pts $f(x+h) = \frac{5}{1-2(x+h)} = \frac{5}{1-2x-2h}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $5(1-2x) - 5(1-2x-2h) = 10h$; 2 pts cancel h to get $\frac{10}{(1-2x-2h)(1-2x)}$; 2 pts let $h \rightarrow 0$. <i>Trap</i> : writing $f(x)+h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.
7	-2	1 pt $g(-1) = -1$; 1 pt setup; 3 pts factoring out $x+1$ to get $x^3 - x^2 + x + 1$, by any valid method. Synthetic or long division by $x+1$ works (on 1, 0, 0, 2, 1, with two zero placeholders). So does grouping: $x^4 + 2x + 1 = (x^4 - 1) + 2(x+1) = (x+1)(x-1)(x^2+1) + 2(x+1) = (x+1)(x^3 - x^2 + x + 1)$. No particular method is required. 3 pts value. <i>Trap</i> : $g(-1) = 1 - 2 = -1$, so subtracting it gives $+1$; the denominator is $x - (-1) = x + 1$. A student who drops a placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} = \sin \frac{\pi}{4}$, so this is the derivative of $\sin$ at $\frac{\pi}{4}$: $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$. <i>Trap</i> : "0/0, DNE" or $-\frac{\sqrt{2}}{2}$; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
8(b)	$\frac{1}{10}$	the derivative of $\sqrt{x}$ at 25: $\frac{1}{2\sqrt{25}} = \frac{1}{10}$; rationalizing the numerator also works. <i>Trap</i> : $\frac{1}{5}$ (forgetting the 2) or 5; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) π (treating π^2 as x^2) or $\pi^2 \cos(\pi/6)$; (b) $f'(2) = 8$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says "for every x ".
10(a)	$18x^{-4} + 2 \cos x$	1 pt rewriting $-6x^{-3}$; 2 pts derivative. <i>Trap</i> : $-6 \cdot (-3) = +18$; the exponent -4 .
10(b)	$\frac{3}{2}x^2 + 3$	1 pt rewriting $\frac{1}{2}x^3 + 3x$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : dividing only the first term by $2x$; differentiating top and bottom separately earns 0.
10(c)	$15x^{3/2}$	1 pt rewriting $6x^{5/2}$; 2 pts derivative. <i>Trap</i> : $x^2\sqrt{x} = x^{5/2}$, not x^1 or $x^{2/2}$; $6 \cdot \frac{5}{2} = 15$.
10(d)	$4x^3 - 8x + \sin x$	1 pt rewriting $x^4 - 4x^2 + 4 - \cos x$; 2 pts derivative. <i>Trap</i> : $(x^2 - 2)^2 \neq x^4 + 4$; the derivative of $-\cos x$ is $+\sin x$.
11	$\sqrt{2} - \frac{3\pi}{2}$	3 pts $f'(x) = 2 \cos x - 6x$; 2 pts value. <i>Trap</i> : $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so $2 \cos \frac{\pi}{4} = \sqrt{2}$; $6 \cdot \frac{\pi}{4} = \frac{3\pi}{2}$. Writing $\frac{3\pi}{4}$ for the second term earns 3.
12	$x = \frac{5\pi}{6}$	2 pts $f'(x) = 2 \cos x + \sqrt{3}$; 2 pts $\cos x = -\frac{\sqrt{3}}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{7\pi}{6}$ also solves $\cos x = -\frac{\sqrt{3}}{2}$ but lies outside $[0, \pi)$; listing it costs 2.
13	IV	6 pts, all or nothing. In IV the solid curve is $f(x) = \frac{x^3}{3} + x^2$ and the dashed curve is $f'(x) = x^2 + 2x$: it is zero exactly where the solid curve turns ($x = -2, 0$), positive where that curve rises, and negative where it falls. Wrong choices: I: shifted: the other curve is zero at $x = \pm 1$, not at the turning points $x = -2$ and $x = 0$; II: right zeros, wrong sign: it is negative where the cubic rises (left of $x = -2$); III: the other curve is zero where the cubic crosses the x -axis ($x = -3$ and $x = 0$), not at the turning point $x = -2$. <i>Trap</i> : assuming the dashed curve is always the derivative; the styles swap between graphs.
14(a)	7	$h'(x) = -2f'(x) + 5g'(x) + 3x^2$, so $h'(1) = -2(3) + 5(2) + (3)$. 1 pt rule, 2 pts correct table entries, 1 pt value. <i>Trap</i> : the columns run $x, g(x), f(x), g'(x), f'(x)$, not the usual order. Using the value columns gives 3, which is $h(1)$; earns 1 (for the rule, if shown). Forgetting the x^3 term's derivative gives 4.
14(b)	-5	$f(4) = 2$ is a number, so $k(x) = 2x^2 - g(x)$ and $k'(x) = 4x - g'(x)$; $k'(-2) = -8 - (-3) = -5$. 2 pts $k'(x)$, 2 pts value. <i>Trap</i> : using $f'(4) = 1$ gives -1 ; earns at most 1.

Item	Answer	Rubric / trap
14(c)	$x = 3$ only	$(f - g)' = f' - g'$, which is 2, 1, 0, -3 at $x = -2, 1, 3, 4$. 2 pts for using $f' - g'$, 2 pts for the answer. <i>Trap:</i> $x = -2$, where $f(x) = g(x)$: there the graphs of f and g meet (so $y = 0$); the graph of y is not flat there. Earns 0.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 6 + 6 + 12 + 5 + 6 + 6 + 12 = 100$.